

What you'll Learn About  
 The Integral Test/P-Series Test/Comparison Test

Integral Test

converges because the area converges

A)  $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots$

$$\lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^2} dx = \lim_{b \rightarrow \infty} \int_1^b x^{-2} dx = \lim_{b \rightarrow \infty} \left[ -x^{-1} \right]_1^b$$

$$= \lim_{b \rightarrow \infty} \left[ -\frac{1}{x} \right]_1^b = \lim_{b \rightarrow \infty} \left[ -\frac{1}{b} - (-1) \right] = 1$$

~~$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{n(n+1)}$~~

Series Diverges b/c area diverges

B)  $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots$  Area Converges

$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$  Do another Test

$$\lim_{b \rightarrow \infty} \int_1^b \frac{1}{x} dx = \lim_{b \rightarrow \infty} \left[ \ln x \right]_1^b = \lim_{b \rightarrow \infty} \left[ \ln b - \ln 1 \right] = \infty$$

Area Diverges

HARMONIC SERIES

Diverges b/c area diverges

C)  $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$

$$\lim_{b \rightarrow \infty} \int_1^b x^{-1/3} dx = \lim_{b \rightarrow \infty} \left[ \frac{3}{2} x^{2/3} \right]_1^b = \lim_{b \rightarrow \infty} \left[ \frac{3}{2} b^{2/3} - \frac{3}{2} \right]$$

= Area Diverges